Sixth Grade Math Curriculum

6th Grade Math Overview

Course Description	Topics at a Glance
The goal of this course is to build on students' past knowledge to develop, understand, and demonstrate math concepts as they prepare for future programs in the middle school. Students will master concepts including, but not limited to, the number system, operations with rational numbers, solving one-step equations and inequalities, rates, ratios, and proportional reasoning. They will also explore geometry via area and volume, and explore statistics and probability.	• Number Theory • Integers • Properties • Solving Equations • Linear Equations

Assessments	Effective Components
• Summative and formative grade level assessments • Classroom assessments • Standardized Tests (ERB)	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Grade Level Expectations

Quantities can be expressed and compared using ratios and rates

Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency

In the real number system, rational numbers have a unique location on the number line and in space

Algebraic expressions can be used to generalize properties of arithmetic

Variables are used to represent unknown quantities within equations and inequalities

Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge

Objects in space and their parts and attributes can be measured and analyzed

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Valwood Graduate Competencies

The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all graduates will be able to demonstrate.

Valwood Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

➢ Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

➢ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

➢ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

➢ Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

➢ Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

➢ Apply transformation to numbers, shapes, functional representations, and data

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Content Area: Mathematics - Sixth Grade
Standard: 1. Number Sense, Properties, and Operations
Valwood Graduates:
Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.
GRADE LEVEL EXPECTATION
Concepts and skills students master:
1. Quantities can be expressed and compared using ratios and rates.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can:	**Inquiry Questions:**
a. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.[1]	1. How are ratios different from fractions?
	2. What is the difference between quantity and number?
b. Apply the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.[2]	**Relevance and Application:**
c. Use ratio and rate reasoning to solve real-world and mathematical problems.[3]	1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions.
i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane.	2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers.
ii. Use tables to compare ratios.	3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks.
iii. Solve unit rate problems including those involving unit pricing and constant speed.[4]	**Nature of Discipline:**
iv. Find a percent of a quantity as a rate per 100.[5]	1. Mathematicians develop simple procedures to express complex mathematical concepts.
v. Solve problems involving finding the whole, given a part and the percent.	2. Mathematicians make sense of problems and persevere in solving them.
vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions.	3. Mathematicians reason abstractly and quantitatively.
vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving	[1] For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." [2] For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." [3] e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
viii. Use ratio reasoning to convert measurement units.[6]	[4] For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? [5] e.g., 30% of a quantity means 30/100 times the quantity. [6] manipulate and transform units appropriately when multiplying or dividing quantities.

Content Area: Mathematics - Sixth Grade	
Standard: 1. Number Sense, Properties, and Operations	

Content Area: Mathematics - Sixth Grade

Standard: 1. Number Sense, Properties, and Operations

Valwood Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL EXPECTATION
Concepts and skills students master:
2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Fluently divide multi-digit numbers using standard algorithms. b. Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. c. Find the greatest common factor of two whole numbers less than or equal to 100. d. Find the least common multiple of two whole numbers less than or equal to 12. e. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.[7] f. Interpret and model quotients of fractions through the creation of story contexts.[8] g. Compute quotients of fractions.[9] h. Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.[10]	**Inquiry Questions:** 1. Why might estimation be better than an exact answer? 2. How do operations with fractions and decimals compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry. **Nature of Discipline:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians model with mathematics. 3. Mathematicians look for and make use of structure. [7] For example, express $36 + 8$ as $4 (9 + 2)$. [8] For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. [9] In general, $(a/b) \div (c/d) = ad/bc$.). [10] How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Content Area: Mathematics - Sixth Grade

Standard: 1. Number Sense, Properties, and Operations

Valwood Graduates:

Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

3. In the real number system, rational numbers have a unique location on the number line and in space.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values.[11]
    i.   Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
b. Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates.[12]
    i.   Describe a rational number as a point on the number line.
    ii. Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line.
    iii. Identify that the opposite of the opposite of a number is the number itself.[13]
    iv. Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
    v. Find and position integers and other rational numbers on a horizontal or vertical number line diagram.
    vi. Find and position pairs of integers and other rational numbers on a coordinate plane.
c. Order and find absolute value of rational numbers.
    i.   Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.[14]
    ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts.[15]
	Inquiry Questions:
	1. Why are there negative numbers?
	2. How do we compare and contrast numbers?
	3. Are there more rational numbers than integers?
	
	Relevance and Application:
	1. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends.
	2. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down.
	
	Nature of Discipline:
	1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations.
	2. Mathematicians construct viable arguments and critique the reasoning of others.
	3. Mathematicians attend to precision.
	
	[11] e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge).
	[12] Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.
	[13] e.g., −(−3) = 3, and that 0 is its own opposite.
	[14] For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.
	[15] For example, write −3 ℃ > −7 ℃ to express the fact that −3 ℃ is warmer than −7 ℃.

iii. Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.[16]

iv. Distinguish comparisons of absolute value from statements about order.[17]

d. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

[16] For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.

[17] For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Valwood Graduate Competencies
The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all graduates will be able to demonstrate.

Valwood Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

➢ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

➢ Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

➢ Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

➢ Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Content Area: Mathematics - Sixth Grade	
Standard: 2. Patterns, Functions, and Algebraic Structures	
Valwood Graduates: Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.	
GRADE LEVEL EXPECTATION **Concepts and skills students master:** 1. Algebraic expressions can be used to generalize properties of arithmetic.	
Evidence Outcomes	**21ˢᵗ Century Skills and Readiness Competencies**

Evidence Outcomes	**21st Century Skills and Readiness Competencies**

Students can:
a. Write and evaluate numerical expressions involving whole-number exponents.
b. Write, read, and evaluate expressions in which letters stand for numbers.
  i. Write expressions that record operations with numbers and with letters standing for numbers.[1]
  ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity.[2]
  iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems.[3]
  iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
c. Apply the properties of operations to generate equivalent expressions.[4]
d. Identify when two expressions are equivalent.[5]
	Inquiry Questions:
	1. If we didn't have variables, what would we use?
	2. What purposes do variable expressions serve?
	3. What are some advantages to being able to describe a pattern using variables?
	4. Why does the order of operations exist?
	5. What other tasks/processes require the use of a strict order of steps?
	
	Relevance and Application:
	1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts.
	2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies.
	
	Nature of Discipline:
	1. Mathematicians use place value to represent many numbers with only ten digits.
	2. Mathematicians construct viable arguments and critique the reasoning of others.
	3. Mathematicians look for and make use of structure.
	4. Mathematicians look for and express regularity in repeated reasoning.
	
	[1] For example, express the calculation "Subtract y from 5" as 5 – y.
	[2] For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
	[3] For example, use the formulas V=s3 and A=6s2 to find the volume and surface area of a cube with sides of length s = 1/2.
	[4] For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
	[5] i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities.

Content Area: Mathematics - Sixth Grade

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

2. Variables are used to represent unknown quantities within equations and inequalities.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? b. Use substitution to determine whether a given number in a specified set makes an equation or inequality true. c. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. i. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. d. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. e. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. f. Show that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	**Inquiry Questions:** 1. Do all equations have exactly one unique solution? Why? 2. How can you determine if a variable is independent or dependent? **Relevance and Application:** 1. Variables allow communication of big ideas with very few symbols. For example, $d=r \times t$ is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and $C=\pi d$ expresses the relationship between circumference and diameter of a circle. 2. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change. **Nature of Discipline:** 1. Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables. 2. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable. 3. Mathematicians model with mathematics. [6] For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

g. Represent and analyze quantitative relationships between dependent and independent variables.
    i. Use variables to represent two quantities in a real-world problem that change in relationship to one another.
    ii. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.
    iii. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.[6]

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Valwood Graduates
The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Valwood education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

➢ Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

➢ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

➢ Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Content Area: Mathematics - Sixth Grade

Standard: 3. Data Analysis, Statistics, and Probability

Valwood Graduates:
  Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.

GRADE LEVEL EXPECTATION
Concepts and skills students master:
  1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
 a. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.[1]
 b. Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
 c. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
 d. Summarize and describe distributions.
   i. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
   ii. Summarize numerical data sets in relation to their context.
     1. Report the number of observations.
     2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.
     3. Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
     4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
	Inquiry Questions:
	 1. Why are there so many ways to describe data?
	 2. When is one data display better than another?
	 3. When is one statistical measure better than another?
	 4. What makes a good statistical question?
	
	 Relevance and Application:
	 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system.
	 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores.
	
	 Nature of Discipline:
	 1. Mathematicians leverage strategic displays to reveal data.
	 2. Mathematicians model with mathematics.
	 3. Mathematicians use appropriate tools strategically.
	 4. Mathematicians attend to precision.
	
	 [1] For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Valwood Graduate Competencies
The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all graduates will be able to demonstrate.

Valwood Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

➢ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

➢ Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

➢ Apply transformation to numbers, shapes, functional representations, and data

➢ Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Content Area: Math - Sixth Grade

Standard: 4. Shape, Dimension, and Geometric Relationships

Valwood Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

1. Objects in space and their parts and attributes can be measured and analyzed.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Develop and apply formulas and procedures for area of plane figures
   i. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.
   ii. Apply these techniques in the context of solving real-world and mathematical problems.
b. Develop and apply formulas and procedures for volume of regular prisms.
   i. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths.
   ii. Show that volume is the same as multiplying the edge lengths of a rectangular prism.
   iii. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
c. Draw polygons in the coordinate plan to solve real-world and mathematical problems.
   i. Draw polygons in the coordinate plane given coordinates for the vertices.
   ii. Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.
d. Develop and apply formulas and procedures for the surface area.
   i. Represent three-dimensional figures using nets made up of rectangles and triangles.
   ii. Use nets to find the surface area of figures.
   iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems.
	Inquiry Questions:
	1. Can two shapes have the same volume but different surface areas? Why?
	2. Can two figures have the same surface area but different volumes? Why?
	3. What does area tell you about a figure?
	4. What properties affect the area of figures?
	
	Relevance and Application:
	1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project.
	2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes.
	
	Nature of Discipline:
	1. Mathematicians realize that measurement always involves a certain degree of error.
	2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning.
	3. Mathematicians make sense of problems and persevere in solving them.
	4. Mathematicians reason abstractly and quantitatively.