VALWOOD

GO BEYOND

PreCalculus Honors Curriculum

PRECALCULUS HONORS COURSE OVERVIEW

Course Description

Mathematics at the Honors Pre-Calculus level focuses on solving and modeling with exponential, logarithmic, parametric and trigonometric functions and their inverses. The complex number system is fully developed. Conic sections, systems of equations solved through matrices, and an introduction to vectors and sequences and series are also included. Limits of functions are taught as a precursor to Calculus. Problem solving, representations, reasoning, communication, and connections within and outside of mathematics underline all of the teaching and learning at the Pre-Calculus level. As an honors course, this course goes beyond the curriculum expectations of the CP Pre-Calculus course offering by increasing the depth and complexity. Students are engaged in dynamic, high-level learning via the Standards for Mathematical Practice. The pace of an advanced course may be faster than that of the CP Pre-Calculus course.

Topics at a Glance

- Exponential and logarithmic functions
- Circle and triangle representation of trigonometric functions
- Trigonometric Identities
- Limits of function
- Trigonometric Equations involving multiple angles and factoring
- Vectors
- Composition of functions and inverses
- Complex number system
- Modeling with trigonometric functions
- Limits of functions
- Conic sections

Assessments

- Teacher Created Assessments
- Assessment task adopted from instructional material

Grade Level Expectations

Standard	Big Ideas for PreCalculus
1. Number Sense, properties, and Operations	1. The complex number system includes real numbers and imaginary numbers 2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations
2. Patterns, Functions, & Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 2. Quantitative relationships in the real world can be modeled and solved using functions 3. Solutions to equations, inequalities and systems of equations are found using a variety of tools 4. Limits and continuity of functions is found.
3. Data Analysis, Statistics & Probability	1. Basic statistics including measures of central tendency, measures of spread, and basic probability are discussed.
4. Shape, Dimension, & Geometric Relationships	1. Fundamental understanding of circular trigonometry can be used in many applications 2. Objects in the plane can be described and analyzed algebraically 3. Objects in the real world can be modeled using geometric concepts

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Valwood Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics – Honors Pre Calculus

Standard: 1. Number Sense, Properties, and Operations

Valwood Graduates:

Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

1. The complex number system includes real numbers and imaginary numbers.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Find the conjugate of a complex number, use conjugates to find moduli and quotients of complex numbers b. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. c. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°. **Using DeMoivres theorem, find powers and roots of complex numbers.** d. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	**Inquiry Questions:** 1. When you extend to a new number systems (e.g., from integers to rational numbers and from rational numbers to real numbers), what properties apply to the extended number system? 2. Are there more complex numbers than real numbers? 3. What is a number system? 4. Why are complex numbers important? **Relevance and Application:** 1. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane. **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

Content Area: Mathematics - Honors Pre Calculus

Standard: 1. Number Sense, Properties, and Operations

Valwood Graduates:

Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes **both in two and three dimensional space.** (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lvert\lvert\boldsymbol{v}\rvert\rvert$, v). b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. c. Solve problems involving velocity and other quantities that can be represented by vectors. d. Add and subtract vectors end-to-end, component-wise, and by the parallelogram rule. 1. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. e. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. f. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. g. Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.	**Inquiry Questions:** 1. How can quantities with size and direction be represented algebraically **and graphically** with vectors? 2. How do the trigonometric ratios connect to the algebraic representation of vectors and its direction angle? **Relevance and Application:** 1. Vectors are used to determine the magnitude and direction of multiple forces acting on an object. 2. Vectors are used to determine the amount of work required to perform a task with a constant force. **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against v (for $c < 0$). c. Find the angle between two vectors and planes. d. Recognize orthogonal vectors and write the representation of a vector orthogonal to a given vector (using cross products), and decompose a vector into two orthogonal vectors.	

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Valwood Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics –Honors Pre Calculus

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Compose functions. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain. e. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. f. Write terms of a sequence and write a formula given a sequence. g. Expand a series and represent terms of a series using summation notation. h. Find the sum of a sequence algebraically and using technology. i. Use the Binomial theorem. j. Graph piecewise-defined functions. k. Defines the limit of a function as the value approached by f(x) as x approaches a given value or infinity with respect to rational functions. l. Finds the limit of a function algebraically and numerically and verifies graphically. m. Understands one-sided limits and continuity of functions.	**Inquiry Questions:** 1. Why are relations and functions represented in multiple ways? 2. What is an inverse? 3. How is "inverse function" most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations? **Relevance and Application:** 1. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments. (PFL) 2. Comprehension of rate of change of a function is important preparation for the study of calculus. 3. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot. 4. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function. 5. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans.

Content Area: Mathematics - Honors Pre Calculus

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:

Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

2. Quantitative relationships in the real world can be modeled and solved using functions.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Model exponential growth and decay functions b. Understand the nature of compound interest and work with the respective formulas c. Write logarithmic equations in exponential form and vice versa d. Understand the relationship between an exponential function and a logarithmic function e. Solve exponential and logarithmic equations f. Identity exponential and logarithmic functions from an equation and graph and identify asymptotes, intercepts, domain and range g. Manipulate expressions using logarithmic properties	**Inquiry Questions:** 1. What is the difference between base b and base e exponential models? 2. How are logarithms used to scale quantities that have an extremely broad range of values? 3. How are exponential properties and logarithmic properties intertwined? 4. How does the nature of the compounding specifically influence the growth factor? **Relevance and Application:** 1. Exponential functions are integral to modeling population growth, compound interest, depreciating car values, half-lives, virus spreading. 2. Logarithmic functions are used to model decibel levels, earthquake magnitudes, pH levels. **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

Content Area: Mathematics - Honors Pre Calculus

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

3. Solutions to equations, inequalities and systems of equations are found using a variety of tools.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Solve a two-by-two and three-by-three system of equations using substitution or elimination. b. Understand the difference between consistent and inconsistent systems of equations. c. Use matrices to solve a system of equations. d. Find the inverse of a matrix. e. Set up a system of equations from a real world application. f. Evaluate a determinant and use Cramer's rule to solve a system of equations. g. Use technology for matrices 3x3 and greater. h. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. i. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. j. Add, subtract, and multiply matrices of appropriate dimensions. k. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. l. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. m. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. n. Work with 2 × 2 matrices as transformations of the plane, and	**Inquiry Questions:** 1. Why do we solve systems using multiple methods ? 2. What are the advantages and disadvantages to each method ? 3. What is the connection between matrices and systems of equations ? **Relevance and Application:** 1. Problems ranging from scheduling airline traffic to controlling traffic flow to routing phone calls over a network involve solving systems of equations with multiple variables. 2. To predict sports records 3. To model college entrance requirements **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

interpret the absolute value of the determinant in terms of area.	

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Valwood Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Valwood Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Content Area: Mathematics - Honors Pre Calculus

Standard: 4. Shape, Dimension, and Geometric Relationships

Valwood Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION: Advanced Pre Calculus

Concepts and skills students master:

1. Fundamental understanding of circular trigonometry can be used in many applications.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Recognize trigonometric function values using the unit circle. i. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. b. Understand the relationship between radians and degrees. i. Convert angle measures between radians and degrees ii. Solve problem situations with angles measures in either mode. iii. Explain when it is most appropriate to use radians or degrees and be able to identify the appropriate mode in problem-solving situations. c. Solve a right triangle using trigonometric ratios. d. Solve problems dealing with circular motion. e. Evaluate the six trigonometric functions for any angle working with the x, y and r components. f. Identify and use reference angles and reference triangles for problem-solving. g. Understand domain and range of the trigonometric functions using the unit circle. h. Determine the arc length in a circle. i. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	**Inquiry Questions:** 1. What is the connection between radians and degrees for measuring angles? 2. Why is the unit circle such an important visual tool for understanding trigonometric function values? 3. What is the underlying meaning for an specific trigonometric function value? 4. Can students understand the three different ways the six trigonometric functions can be defined: as the ratio of two sides of a right triangle, as coordinates of a point x,y , in the plane and, its distance r from the origin and as functions of any real number? **Relevance and Application:** 1. Trigonometric ratios are used in navigation, building and engineering. 2. To use ancient calendars. 3. Angles are used to find distances around a circle, or how fast a point moves around a circle. **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

Content Area: Mathematics - Honors Pre Calculus

Standard: 4. Shape, Dimension, and Geometric Relationships

Valwood Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

2. Objects in the plane can be described and analyzed algebraically.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Identify equations of conic sections and graph them on a Cartesian coordinate plane b. Identify key characteristics for each conic section, including focus/foci, vertex/vertices, center, asymptotes c. Convert between standard form and general form for each conic section d. Model practical applications involving conic sections e. Understand the geometric definitions for each conic section in terms of the locus f. Graph rotated conics by removing the xy term and by using technology. g. Write conics in polar form and graph h. Graph parametric equations by hand and using technology i. Convert equations in rectangular form to parametric and vice versa j. Model practical applications involving parametric equations	**Inquiry Questions:** 1. What are the fundamental differences in the equations for each conic section 2. How does the position of the focus/foci, vertex/vertices, center, asymptotes affect the graphical nature of each curve **Relevance and Application:** 1. The mathematics of conic sections is present in the movement of planets, bridge and tunnel construction, navigational systems used to keep track of a ship's location and the manufacture of lenses for telescopes 2. To model gears in machinery **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.

Content Area: Mathematics – Honors Pre Calculus

Standard: 4. Shape, Dimension, and Geometric Relationships

Valwood Graduates:

Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

3. Objects in the real world can be modeled using geometric concepts.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Apply the trigonometric values that they learn from the different representations (unit circle, right triangle and reference angles) to graph the function in the Cartesian coordinate plane b. Identify and understand key characteristics of trigonometric functions (period, vertical shift, amplitude and phase shift). c. Find an equation for and analyze an object in simple harmonic and damped motion. d. Find the domain and range of inverse trigonometric functions and use restrictions on the domains of the inverse trigonometric functions to find values of the inverse functions. e. Use trigonometric identities to make algebraic substitutions to simplify and verify trigonometric identities. f. Use trigonometric functions, Pythagorean Theorem, Law of Sines and Law of Cosines and Heron's formula to solve practical problems. g. Solve trigonometric equations using sum and difference identities, half-angle identities and double-angle identities	**Inquiry Questions:** 1. How the parts of the trigonometric equation a,b,c and d affect the nature of the trigonometric curve 2. How can we model periodic applications using sinusoidal function 3. How does making the trigonometric functions one-to-one enables us to find an inverse that is a function and how are the domains restricted to accomplish this 4. How are trigonometric identities fundamentally different from conditional equations 5. What algebraic skills are necessary to verify trigonometric identities **Relevance and Application:** 1. Trigonometric functions are used to model periodic quantities such as tidal flows, weather patterns, daylight hours, points moving in circular motion, sound waves, light waves 2. Law of Sines and Cosines are used to solve problems of navigation, distances, surveying **Nature of Discipline:** 1. Mathematicians apply math concepts to real world problem solving. 2. Mathematicians communicate their reasoning used to solve problems. 3. Mathematicians are able to connect concept and process to effectively solve problems.