Algebra II Honors Curriculum

Algebra II Honors Course Overview

Course Description

Honors Algebra 2 emphasizes the structure of algebra. The students will study complex numbers, relations and functions, solutions to polynomial, radical, rational, exponential and logarithmic functions. Students start on linear equations, systems of equations and inequalities, and matrices and determinants. A great deal of time will be spent working with mathematical functions including quadratic, polynomial, rational, exponential, and logarithmic. Conic sections and an introduction to trigonometry will be covered toward the end of the course. They will apply their studies to develop understandings of how these topics relate to one another through the Standards for Mathematical Practice.

Topics at a Glance

- Complex number system
- Properties of expressions, use of and solving equations and functions for: Exponential and logarithmic, rational, radical, polynomial.
- Conic Sections
- Interpreting and comparing functions, relations, and transformations
- Systems of linear equations and inequalities
- Systems of non-linear equations and inequalities
- Sequenses and Series
- Matricies and determinants

Assessments

- Teacher created assessments
- Class assessments
- Assessments adopted from course materials

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade Level Expectations

Standard	Big Ideas for Algebra 2
1. Number Sense, properties, and operations	1. The complex number system includes real numbers and imaginary numbers
2. Patterns, Functions, & Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables 2. Quantitative relationships in the real world can be modeled and solved using functions 3. Expressions can be represented in multiple, equivalent forms 4. Solutions to equations, inequalities and systems of equations are found using a variety of tools
3. Data Analysis, Statistics, & Probability	
4. Shape, Dimension, & Geometric Relationships	1. Objects in the plane can be described and analyzed algebraically 2. Objects in the real world can be modeled using geometric concepts

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Valwood Graduates

The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

➢ Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

➢ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

➢ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

➢ Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning

➢ Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

➢ Apply transformation to numbers, shapes, functional representations, and data

Content Area: Mathematics – Honors Algebra 2
Standard: 1. Number Sense, Properties, and Operations
Valwood Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.
GRADE LEVEL EXPECTATION **Concepts and skills students master:** 1. The complex number system includes real numbers and imaginary numbers.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Perform arithmetic operations with complex numbers.
   i.   Define the complex number i such that $i^2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real numbers.
   ii. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
b. Use complex numbers in polynomial identities and equations.
c. Solve quadratic equations with real coefficients that have complex solutions.
	Inquiry Questions:
	1. When you extend to a new number systems (e.g., from integers to rational numbers and from rational numbers to real numbers), what properties apply to the extended number system?
	2. Are there more complex numbers than real numbers?
	3. What is a number system?
	4. Why are complex numbers important?

	Relevance and Application: 1. Complex numbers have applications in fields such as chaos theory and fractals. The familiar image of the Mandelbrot fractal is the Mandelbrot set graphed on the complex plane.
	Nature of Discipline: 1. Mathematicians build a deep understanding of quantity, ways of representing numbers, and relationships among numbers and number systems. 2. Mathematicians look for and make use of structure. 3. Mathematicians look for and express regularity in repeated reasoning.

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Valwood Graduates
The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

➢ Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

➢ Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

➢ Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

➢ Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

---

Content Area: Mathematics – Honors Algebra 2

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
  Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.

GRADE LEVEL EXPECTATION

Concepts and skills students master:
  1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
 a. Interpret functions that arise in applications in terms of the context.
    i. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
    ii. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
    iii. Calculate and interpret the average rate of change[5] of a function over a specified interval. Estimate the rate of change from a graph.
 	
 b. Analyze functions using different representations.
    i. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases
    ii. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    iii. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    iv. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	Inquiry Questions:
	 1. Why are relations and functions represented in multiple ways?
	 2. How can a table, graph, and function notation be used to explain how one function family is different from and/or similar to another?
	 3. What is an inverse?
	 4. How is "inverse function" most likely related to addition and subtraction being inverse operations and to multiplication and division being inverse operations?
	 5. How are patterns and functions similar and different?
	 6. How could you visualize a function with four variables, such as $x^2 + y^2 + z^2 + w^2 = 1$?
	 7. How do symbolic transformations affect an equation, inequality, or expression?
	
	 Relevance and Application:
	
	 1. Knowledge of how to interpret rate of change of a function allows investigation of rate of return and time on the value of investments.
	 2. Comprehension of rate of change of a function is important preparation for the study of calculus.
	 3. The ability to analyze a function for the intercepts, asymptotes, domain, range, and local and global behavior provides insights into the situations modeled by the function. For example, epidemiologists could compare the rate of flu infection among people who received flu shots to the rate of flu infection among people who did not receive a flu shot to gain insight into the effectiveness of the flu shot.

      v.   Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

c. Build a function that models a relationship between two quantities.
      i.   Write a function that describes a relationship between two quantities.

d. Build new functions from existing functions.
      i.   Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k, [9] and find the value of k given the graphs.
          1.   Combine standard function types using arithmetic operations.
      ii.   Experiment with cases and illustrate an explanation of the effects on the graph using technology.
      iii.   Find inverse functions.

e. Extend the domain of trigonometric functions using the unit circle.
      i.   Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
      ii.   Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

4. The exploration of multiple representations of functions develops a deeper understanding of the relationship between the variables in the function.
5. The understanding of the relationship between variables in a function allows people to use functions to model relationships in the real world such as compound interest, population growth and decay, projectile motion, or payment plans.
6. Comprehension of slope, intercepts, and common forms of linear equations allows easy retrieval of information from linear models such as rate of growth or decrease, an initial charge for services, speed of an object, or the beginning balance of an account.
7. *Understanding sequences is important preparation for calculus. Sequences can be used to represent functions including e^x, e^{x^2}, sin x, and cos x.*
8. How do symbolic transformations affect an equation, inequality, or expression?

Nature of Discipline:
1. Mathematicians use multiple representations of functions to explore the properties of functions and the properties of families of functions.
2. Mathematicians model with mathematics.
3. Mathematicians use appropriate tools strategically.
4. Mathematicians look for and make use of structure.

Content Area: Mathematics -Honors Algebra 2

Standard: 2 Patterns, Functions, and Algebraic Structures

Valwood Graduates:
  Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

GRADE LEVEL EXPECTATION
Concepts and skills students master:
  2. Quantitative relationships in the real world can be modeled and solved using functions.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Construct and compare linear, quadratic, and exponential models and solve problems. i. For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. b. Model periodic phenomena with trigonometric functions. c. hoose the trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	**Inquiry Questions:** 1. Why do we classify functions? 2. What phenomena can be modeled with particular functions? 3. Which financial applications can be modeled with exponential functions? Linear functions? 4. What elementary function or functions best represent a given scatter plot of two-variable data? 5. How much would today's purchase cost tomorrow?
	Relevance and Application: 1. The understanding of the qualitative behavior of functions allows interpretation of the qualitative behavior of systems modeled by functions such as time-distance, population growth, decay, heat transfer, and temperature of the ocean versus depth. 2. The knowledge of how functions model real-world phenomena allows exploration and improved understanding of complex systems such as how population growth may affect the environment , how interest rates or inflation affect a personal budget, how stopping distance is related to reaction time and velocity, and how volume and temperature of a gas are related. 3. Biologists use polynomial curves to model the shapes of jaw bone fossils. They analyze the polynomials to find potential evolutionary relationships among the species. 4. Physicists use basic linear and quadratic functions to model the motion of projectiles.
	Nature of Discipline: 1. Mathematicians use their knowledge of functions to create accurate models of complex systems. 2. Mathematicians use models to better understand systems and make predictions about future systemic behavior.

	3. Mathematicians reason abstractly and quantitatively.
	4. Mathematicians construct viable arguments and critique the reasoning of others.
	5. Mathematicians model with mathematics.

Content Area: Mathematics -Honors Algebra 2

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.

GRADE LEVEL EXPECTATION
Concepts and skills students master:
3. Expressions can be represented in multiple, equivalent forms.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Interpret the structure of expressions.
    i. Interpret expressions that represent a quantity in terms of its context.
      1. Interpret parts of an expression, such as terms, factors, and coefficients.
      2. Interpret complicated expressions by viewing one or more of their parts as a single entity.
   ii. Use the structure of an expression to identify ways to rewrite it.
b. Write expressions in equivalent forms to solve problems.
   i. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
c. Perform arithmetic operations on polynomials.
   i. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
d. Understand the relationship between zeros and factors of polynomials.
   i. State and apply the Remainder Theorem.
   ii. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
e. Use polynomial identities to solve problems.
   i. Prove polynomial identities[18] and use them to describe numerical relationships.
f. Rewrite rational expressions.
	Inquiry Questions:
	1. When is it appropriate to simplify expressions?
	2. The ancient Greeks multiplied binomials and found the roots of quadratic equations without algebraic notation. How can this be done?
	
	Relevance and Application:
	1. The simplification of algebraic expressions and solving equations are tools used to solve problems in science. Scientists represent relationships between variables by developing a formula and using values obtained from experimental measurements and algebraic manipulation to determine values of quantities that are difficult or impossible to measure directly such as acceleration due to gravity, speed of light, and mass of the earth.
	2. The manipulation of expressions and solving formulas are techniques used to solve problems in geometry such as finding the area of a circle, determining the volume of a sphere, calculating the surface area of a prism, and applying the Pythagorean Theorem.
	
	Nature of Discipline:
	1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions.
	2. Mathematicians construct viable arguments and critique the reasoning of others.
	3. Mathematicians model with mathematics.
	4. Mathematicians look for and express regularity in repeated reasoning.

g. Rewrite simple rational expressions in different forms.	

Content Area: Mathematics – Honors Algebra 2

Standard: 2. Patterns, Functions, and Algebraic Structures

Valwood Graduates:

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

GRADE LEVEL EXPECTATION

Concepts and skills students master:

4. Solutions to equations, inequalities and systems of equations are found using a variety of tools.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Create equations that describe numbers or relationships.
    i. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales.
    ii. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
    iii. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
b. Understand solving equations as a process of reasoning and explain the reasoning.
    i. Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
c. Solve equations and inequalities in one variable.
    i. Solve quadratic equations in one variable.
        1. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
	Inquiry Questions:
	1. What are some similarities in solving all types of equations?
	2. Why do different types of equations require different types of solution processes?
	3. Can computers solve algebraic problems that people cannot solve? Why?
	4. How are order of operations and operational relationships important when solving multivariable equations?
	
	Relevance and Application:
	1. Linear programming allows representation of the constraints in a real-world situation identification of a feasible region and determination of the maximum or minimum value such as to optimize profit, or to minimize expense.
	2. Effective use of graphing technology helps to find solutions to equations or systems of equations.
	
	Nature of Discipline:
	1. Mathematics involves visualization.
	2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning.
	3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
	4. Mathematicians use appropriate tools strategically. (MP)

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Valwood Graduates

The Valwood graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Valwood Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

➢ Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts

➢ Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

➢ Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

---

Content Area: Mathematics – Honors Algebra 2

Standard: 3. Data Analysis, Statistics, and Probability

Valwood Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.

GRADE LEVEL EXPECTATION
Concepts and skills students master:
1. Visual displays and summary condense the information in data sets into usable knowledge.

Evidence Outcomes	21st Century Skills and Readiness Competencies

Students can:
a. Summarize, represent, and interpret data on a single count or measurement variable.
   i. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate.
   ii. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
	Inquiry Questions:
	1. What makes data meaningful or actionable?
	2. Why should attention be paid to an unexpected outcome?
	3. How can summary statistics or data displays be accurate but misleading?

	Relevance and Application: 1. Facility with data organization, summary, and display allows the sharing of data efficiently and collaboratively to answer important questions such as is the climate changing, how do people think about ballot initiatives in the next election, or is there a connection between cancers in a community?
	Nature of Discipline: 1. Mathematicians create visual and numerical representations of data to reveal relationships and meaning hidden in the raw data. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians model with mathematics. 4. Mathematicians use appropriate tools strategically.

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates
The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

➢ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

➢ Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

➢ Apply transformation to numbers, shapes, functional representations, and data

➢ Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

➢ Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

---